Public Schools of North Carolina
State Board of Education | Department of Public Instruction

8th Grade Mathematics ● Unpacked Content

For the new Common Core standards that will be effective in all North Carolina schools in the 2012-13 School Year.

This document is designed to help North Carolina educators teach the Common Core (Standard Course of Study). NCDPI staff are continually updating and improving these tools to better serve teachers.

What is the purpose of this document?
To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do.

What is in the document?
Descriptions of what each standard means a student will know, understand and be able to do. The "unpacking" of the standards done in this document is an effort to answer a simple question "What does this standard mean that a student must know and be able to do?" and to ensure the description is helpful, specific and comprehensive for educators.

How do I send Feedback?
We intend the explanations and examples in this document to be helpful and specific. That said, we believe that as this document is used, teachers and educators will find ways in which the unpacking can be improved and made ever more useful. Please send feedback to us at feedback@dpi.state.nc.us and we will use your input to refine our unpacking of the standards. Thank You!

Just want the standards alone?
You can find the standards alone at www.corestandards.org .

The Number System 8.NS

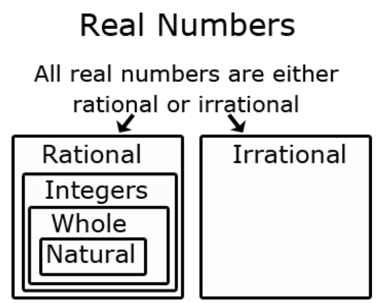
Common Core Cluster

Know that there are numbers that are not rational, and approximate them by rational numbers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **Real Numbers, Irrational numbers, Rational numbers, Integers, Whole numbers, Natural numbers, radical, radicand, square roots, perfect squares, cube roots, terminating decimals, repeating decimals, truncate**

<table>
<tr><th>Common Core Standard</th><th>Unpacking
What does this standard mean that a student will know and be able to do?</th></tr>
<tr><td>8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.</td><td>

8.NS.1 Students understand that Real numbers are either rational or irrational. They distinguish between rational and irrational numbers, recognizing that any number that can be expressed as a fraction is a rational number. The diagram below illustrates the relationship between the subgroups of the real number system.



Students recognize that the decimal equivalent of a fraction will either terminate or repeat. Fractions that terminate will have denominators containing only prime factors of 2 and/or 5. This understanding builds on work in 7th grade when students used long division to distinguish between repeating and terminating decimals.

Students convert repeating decimals into their fraction equivalent using patterns or algebraic reasoning.
One method to find the fraction equivalent to a repeating decimal is shown below.

<u>Example 1:</u>
Change $0.\overline{4}$ to a fraction.

- Let $x = 0.444444\ldots..$
- Multiply both sides so that the repeating digits will be in front of the decimal. In this example, one digit repeats so both sides are multiplied by 10, giving $10x = 4.4444444\ldots.$

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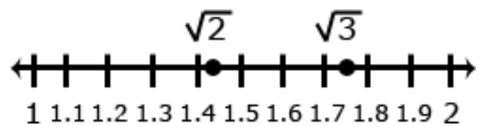
	• Subtract the original equation from the new equation. $$\begin{array}{r} 10x = 4.4444444\ldots. \\ \underline{-\,x = 0.444444\ldots..} \\ 9x = 4 \end{array}$$ • Solve the equation to determine the equivalent fraction. $$\frac{9x}{9} = \frac{4}{9}$$ $$x = \frac{4}{9}$$ Additionally, students can investigate repeating patterns that occur when fractions have denominators of 9, 99, or 11. <u>Example 2:</u> $\frac{4}{9}$ is equivalent to $0.\overline{4}$, $\frac{5}{9}$ is equivalent to $0.\overline{5}$, etc.
8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	**8.NS.2** Students locate rational and irrational numbers on the number line. Students compare and order rational and irrational numbers. Students also recognize that square roots may be negative and written as $-\sqrt{28}$. <u>Example 1:</u> Compare $\sqrt{2}$ and $\sqrt{3}$  *Solution:* Statements for the comparison could include: $\sqrt{2}$ and $\sqrt{3}$ are between the whole numbers 1 and 2 $\sqrt{3}$ is between 1.7 and 1.8 $\sqrt{2}$ is less than $\sqrt{3}$ Additionally, students understand that the value of a square root can be approximated between integers and that non-perfect square roots are irrational. <u>Example 2:</u> Find an approximation of $\sqrt{28}$ • Determine the perfect squares $\sqrt{28}$ is between, which would be 25 and 36. • The square roots of 25 and 36 are 5 and 6 respectively, so we know that $\sqrt{28}$ is between 5 and 6. • Since 28 is closer to 25, an estimate of the square root would be closer to 5. One method to get an estimate is to divide 3 (the distance between 25 and 28) by 11 (the distance between the perfect squares of 25 and 36) to get 0.27. • The estimate of $\sqrt{28}$ would be 5.27 (the actual is 5.29).

Expressions and Equations 8.EE

Common Core Cluster

Work with radicals and integer exponents.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **laws of exponents, power, perfect squares, perfect cubes, root, square root, cube root, scientific notation, standard form of a number.** Students should also be able to read and use the symbol: ±

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	**8.EE.1** In 6th grade, students wrote and evaluated simple numerical expressions with whole number exponents (ie. $5^3 = 5 \bullet 5 \bullet 5 = 125$). Integer (positive and negative) exponents are further developed to generate equivalent numerical expressions when multiplying, dividing or raising a power to a power. Using numerical bases and the laws of exponents, students generate equivalent expressions. Students understand: • Bases must be the same before exponents can be added, subtracted or multiplied. (Example 1) • Exponents are subtracted when like bases are being divided (Example 2) • A number raised to the zero (0) power is equal to one. (Example 3) • Negative exponents occur when there are more factors in the denominator. These exponents can be expressed as a positive if left in the denominator. (Example 4) • Exponents are added when like bases are being multiplied (Example 5) • Exponents are multiplied when an exponents is raised to an exponent (Example 6) • Several properties may be used to simplify an expression (Example 7) <u>Example 1:</u> $\frac{2^3}{5^2} = \frac{8}{25}$ <u>Example 2:</u> $\frac{2^2}{2^6} = 2^{2-6} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$ <u>Example 3:</u> $6^0 = 1$ Students understand this relationship from examples such as $\frac{6^2}{6^2}$. This expression could be simplified as $\frac{36}{36} = 1$. Using the laws of exponents this expression could also be written as $6^{2-2} = 6^0$. Combining these gives $6^0 = 1$.

	Example 4: $\frac{3^{-2}}{2^4} = 3^{-2} \text{ x } \frac{1}{2^4} = \frac{1}{3^2} \text{ x } \frac{1}{2^4} = \frac{1}{9} \text{ x } \frac{1}{16} = \frac{1}{144}$ Example 5: $(3^2)(3^4) = (3^{2+4}) = 3^6 = 729$ Example 6: $(4^3)^2 = 4^{3\text{x}2} = 4^6 = 4{,}096$ Example 7: $\frac{(3^2)^4}{(3^2)(3^3)} = \frac{3^{2x4}}{3^{2+3}} = \frac{3^8}{3^5} = 3^{8-5} = 3^3 = 27$
8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.	**8.EE.2** Students recognize perfect squares and cubes, understanding that non-perfect squares and non-perfect cubes are irrational. Students recognize that squaring a number and taking the square root √ of a number are inverse operations; likewise, cubing a number and taking the cube root $\sqrt[3]{}$ are inverse operations. Example 1: $4^2 = 16$ and $\sqrt{16} = \pm 4$ NOTE: $(-4)^2 = 16$ while $-4^2 = -16$ since the negative is not being squared. This difference is often problematic for students, especially with calculator use. Example 2: $\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27}$ and $\sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3}$ NOTE: there is no negative cube root since multiplying 3 negatives would give a negative. This understanding is used to solve equations containing square or cube numbers. Rational numbers would have perfect squares or perfect cubes for the numerator and denominator. In the standard, the value of p for square root and cube root equations must be positive. Example 3: Solve: $x^2 = 25$ *Solution:* $\sqrt{x^2} = \pm\sqrt{25}$ $x = \pm 5$ NOTE: There are two solutions because 5 • 5 and -5 • -5 will both equal 25.

	Example 4: Solve: $x^2 = \frac{4}{9}$ *Solution:* $\sqrt{x^2} = \pm\sqrt{\frac{4}{9}}$ $x = \pm\frac{2}{3}$ Example 5: Solve: $x^3 = 27$ *Solution:* $\sqrt[3]{x} = \sqrt[3]{27}$ $x = 3$ Example 6: Solve: $x^3 = \frac{1}{8}$ *Solution:* $\sqrt[3]{x} = \sqrt[3]{\frac{1}{8}}$ $x = \frac{1}{2}$ Students understand that in geometry the square root of the area is the length of the side of a square and a cube root of the volume is the length of the side of a cube. Students use this information to solve problems, such as finding the perimeter. Example 7: What is the side length of a square with an area of 49 ft^2? *Solution:* $\sqrt{49} = 7$ ft. The length of one side is 7 ft.
8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*	**8.EE.3** Students use scientific notation to express very large or very small numbers. Students compare and interpret scientific notation quantities in the context of the situation, recognizing that if the exponent increases by one, the value increases 10 times. Likewise, if the exponent decreases by one, the value decreases 10 times. Students solve problems using addition, subtraction or multiplication, expressing the answer in scientific notation. Example 1: Write 75,000,000,000 in scientific notation. *Solution:* 7.5×10^{10} Example 2: Write 0.0000429 in scientific notation. *Solution:* 4.29×10^{-5}

	<u>Example 3:</u> Express 2.45×10^5 in standard form. *Solution:* 245,000 <u>Example 4:</u> How much larger is 6×10^5 compared to 2×10^3 *Solution:* 300 times larger since 6 is 3 times larger than 2 and 10^5 is 100 times larger than 10^3. <u>Example 5:</u> Which is the larger value: 2×10^6 or 9×10^5? *Solution:* 2×10^6 because the exponent is larger
8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	**8.EE.4** Students understand scientific notation as generated on various calculators or other technology. Students enter scientific notation using E or EE (scientific notation), * (multiplication), and ^ (exponent) symbols. <u>Example 1:</u> 2.45E+23 is 2.45×10^{23} and 3.5E-4 is 3.5×10^{-4} (NOTE: There are other notations for scientific notation depending on the calculator being used.) Students add and subtract with scientific notation. <u>Example 2:</u> In July 2010 there were approximately 500 million facebook users. In July 2011 there were approximately 750 million facebook users. How many more users were there in 2011. Write your answer in scientific notation. *Solution:* Subtract the two numbers: 750,000,000 - 500,000,000 = 250,000,000 → 2.5×10^8 Students use laws of exponents to multiply or divide numbers written in scientific notation, writing the product or quotient in proper scientific notation. <u>Example 3:</u> $(6.45 \times 10^{11})(3.2 \times 10^4) = (6.45 \times 3.2)(10^{11} \times 10^4)$ *Rearrange factors* $= 20.64 \times 10^{15}$ *Add exponents when multiplying powers of 10* $= 2.064 \times 10^{16}$ *Write in scientific notation* <u>Example 4:</u> $\dfrac{3.45 \times 10^5}{6.7 \times 10^{-2}} \quad \dfrac{6.3}{1.6} \times 10^{5-(-2)}$ *Subtract exponents when dividing powers of 10* $= 0.515 \times 10^7$ *Write in scientific notation* $= 5.15 \times 10^6$ <u>Example 5:</u> $(0.0025)(5.2 \times 10^4) = (2.5 \times 10^{-3})(5.2 \times 10^5)$ *Write factors in scientific notation* $= (2.5 \times 5.2)(10^{-3} \times 10^5)$ *Rearrange factors* $= 13 \times 10^2$ *Add exponents when multiplying powers of 10* $= 1.3 \times 10^3$ *Write in scientific notation*

	Example 6: The speed of light is 3×10^8 meters/second. If the sun is 1.5×10^{11} meters from earth, how many seconds does it take light to reach the earth? Express your answer in scientific notation. *Solution*: 5×10^2 (light)(x) = sun, where x is the time in seconds $(3 \times 10^8)x = 1.5 \times 10^{11}$ $\frac{1.5 \times 10^{11}}{3 \times 10^8}$ Students understand the magnitude of the number being expressed in scientific notation and choose an appropriate corresponding unit. Example 7: 3×10^8 is equivalent to 300 million, which represents a large quantity. Therefore, this value will affect the unit chosen.

Expressions and Equations — 8.EE

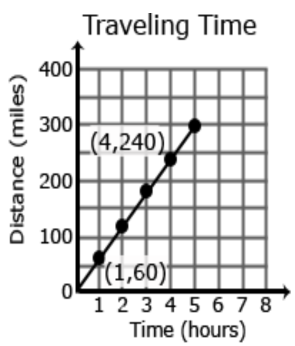
Common Core Cluster

Understand the connections between proportional relationships, lines, and linear equations.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **unit rate, proportional relationships, slope, vertical, horizontal, similar triangles, *y*-intercept**

<table>
<tr><th>Common Core Standard</th><th>Unpacking
What does this standard mean that a student will know and be able to do?</th></tr>
<tr><td>8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</td><td>8.EE.5 Students build on their work with unit rates from 6th grade and proportional relationships in 7th grade to compare graphs, tables and equations of proportional relationships. Students identify the unit rate (or slope) in graphs, tables and equations to compare two proportional relationships represented in different ways.

<u>Example 1:</u>
Compare the scenarios to determine which represents a greater speed. Explain your choice including a written description of each scenario. Be sure to include the unit rates in your explanation.

Scenario 1:


Scenario 2:
$y = 55x$
x is time in hours
y is distance in miles

Solution: Scenario 1 has the greater speed since the unit rate is 60 miles per hour. The graph shows this rate since 60 is the distance traveled in one hour. Scenario 2 has a unit rate of 55 miles per hour shown as the coefficient in the equation.

Given an equation of a proportional relationship, students draw a graph of the relationship. Students recognize that the unit rate is the coefficient of x and that this value is also the slope of the line.</td></tr>
<tr><td>8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a</td><td>8.EE.6 Triangles are similar when there is a constant rate of proportionality between them. Using a graph, students construct triangles between two points on a line and compare the sides to understand that the slope (ratio of rise to run) is the same between any two points on a line.</td></tr>
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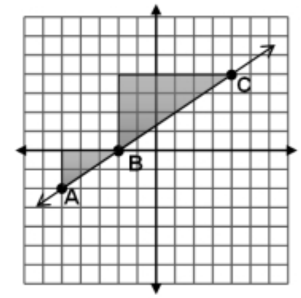
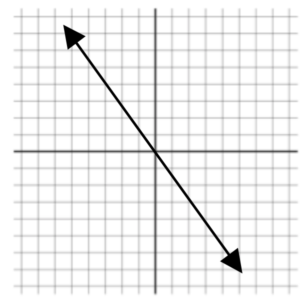
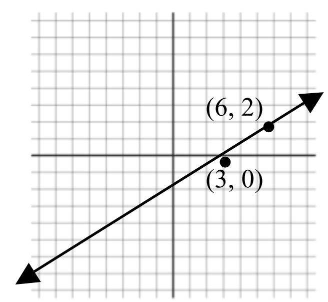
non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	<u>Example 1:</u> The triangle between A and B has a vertical height of 2 and a horizontal length of 3. The triangle between B and C has a vertical height of 4 and a horizontal length of 6. The simplified ratio of the vertical height to the horizontal length of both triangles is 2 to 3, which also represents a slope of $\frac{2}{3}$ for the line, indicating that the triangles are similar.  Given an equation in slope-intercept form, students graph the line represented. Students write equations in the form $y = mx$ for lines going through the origin, recognizing that m represents the slope of the line. <u>Example 2:</u> Write an equation to represent the graph to the right.  *Solution:* $y = -\frac{3}{2}x$ Students write equations in the form $y = mx + b$ for lines not passing through the origin, recognizing that m represents the slope and b represents the y-intercept.  *Solution:* $y = \frac{2}{3}x - 2$

Expressions and Equations 8.EE

Common Core Cluster

Analyze and solve linear equations and pairs of simultaneous linear equations.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **intersecting, parallel lines, coefficient, distributive property, like terms, substitution, system of linear equations**

<table>
<tr><th>Common Core Standard</th><th>Unpacking
What does this standard mean that a student will know and be able to do?</th></tr>
<tr><td>8.EE.7 Solve linear equations in one variable.
a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</td><td>8.EE.7 Students solve one-variable equations including those with the variables being on both sides of the equals sign. Students recognize that the solution to the equation is the value(s) of the variable, which make a true equality when substituted back into the equation. Equations shall include rational numbers, distributive property and combining like terms.

<u>Example 1:</u>
Equations have one solution when the variables do not cancel out. For example, $10x – 23 = 29 – 3x$ can be solved to $x = 4$. This means that when the value of x is 4, both sides will be equal. If each side of the equation were treated as a linear equation and graphed, the solution of the equation represents the coordinates of the point where the two lines would intersect. In this example, the ordered pair would be (4, 17).

$10 \bullet 4 – 23 = 29 – 3 \bullet 4$
$40 – 23 = 29 – 12$
$17 = 17$

<u>Example 2:</u>
Equations having no solution have variables that will cancel out and constants that are not equal. This means that there is not a value that can be substituted for x that will make the sides equal.
$-x + 7 – 6x = 19 – 7x$ Combine like terms
$-7x + 7 = 19 – 7x$ Add 7x to each side
$7 \neq 19$

This solution means that no matter what value is substituted for x the final result will never be equal to each other.

If each side of the equation were treated as a linear equation and graphed, the lines would be parallel.</td></tr>
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<u>Example 3:</u>
An equation with infinitely many solutions occurs when both sides of the equation are the same. Any value of x will produce a valid equation. For example the following equation, when simplified will give the same values on both sides.
$-\frac{1}{2}(36a - 6) = \frac{3}{4}(4 - 24a)$
$-18a + 3 = 3 - 18a$

If each side of the equation were treated as a linear equation and graphed, the graph would be the same line.

Students write equations from verbal descriptions and solve.
<u>Example 4:</u>
Two more than a certain number is 15 less than twice the number. Find the number.
Solution:
$n + 2 = 2n - 15$
$17 = n$
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<td>
8.EE.8 Analyze and solve pairs of simultaneous linear equations.
a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
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8.EE.8 Systems of linear equations can also have one solution, infinitely many solutions or no solutions. Students will discover these cases as they graph systems of linear equations and solve them algebraically.

Students graph a system of two linear equations, recognizing that the ordered pair for the point of intersection is the x-value that will generate the given y-value for both equations. Students recognize that graphed lines with one point of intersection (different slopes) will have one solution, parallel lines (same slope, different y-intercepts) have no solutions, and lines that are the same (same slope, same y-intercept) will have infinitely many solutions.

By making connections between algebraic and graphical solutions and the context of the system of linear equations, students are able to make sense of their solutions. Students need opportunities to work with equations and context that include whole number and/or decimals/fractions. Students define variables and create a system of linear equations in two variables

<u>Example 1:</u>

1. Plant A and Plant B are on different watering schedules. This affects their rate of growth. Compare the growth of the two plants to determine when their heights will be the same.

Solution:
Let W = number of weeks
Let H = height of the plant after W weeks
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b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.
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<td>
c. Solve real-world and mathematical problems leading to two linear equations in two variables. For
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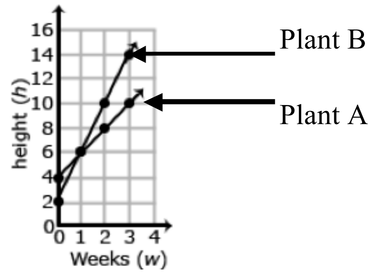
example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Plant A		
W	H	
0	4	(0, 4)
1	6	(1, 6)
2	8	(2, 8)
3	10	(3, 10)

Plant B		
W	H	
0	2	(0, 2)
1	6	(1, 6)
2	10	(2, 10)
3	14	(3, 14)

2. Based on the coordinates from the table, graph lines to represent each plant.

Solution:



3. Write an equation that represents the growth rate of Plant A and Plant B.

Solution: Plant A $H = 2W + 4$

Plant B $H = 4W + 2$

4. At which week will the plants have the same height?

Solution:

$2W + 4 = 4W + 2$ *Set height of Plant A equal to height of Plant B*

$2W - 2W + 4 = 4W - 2W + 2$ *Solve for W*

$4 = 2W + 2$

$4 - 2 = 2W + 2 - 2$

$\frac{2}{2} = \frac{2W}{2}$

$1 = W$

After one week, the height of Plant A and Plant B are both 6 inches.

Check: $2(1) + 4 = 4(1) + 2$

$2 + 4 = 4 + 2$

$6 = 6$

Given two equations in slope-intercept form (Example 1) or one equation in standard form and one equation in slope-intercept form, students use substitution to solve the system.

Example 2:
Solve: Victor is half as old as Maria. The sum of their ages is 54. How old is Victor?

Solution: Let v = Victor's age
Let m = Maria's age

$$\begin{cases} v + m = 54 \\ v = \frac{1}{2} m \end{cases}$$

$\frac{1}{2} m + m = 54$ *Substitute ½m for v in the first equation*

$1\frac{1}{2} m = 54$

$m = 36$

If Maria is 36, then substitute 36 into $v + m = 54$ to find Victor's age of 18.

Note: Students are not expected to change linear equations written in standard form to slope-intercept form or solve systems using elimination.

For many real world contexts, equations may be written in standard form. Students are not expected to change the standard form to slope-intercept form. However, students may generate ordered pairs recognizing that the values of the ordered pairs would be solutions for the equation. For example, in the equation above, students could make a list of the possible ages of Victor and Maria that would add to 54. The graph of these ordered pairs would be a line with all the possible ages for Victor and Maria.

Victor	Maria
20	34
10	44
50	4
29	25

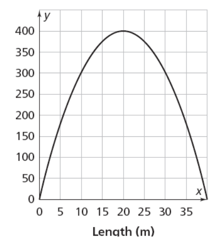
Functions 8.F

Common Core Cluster

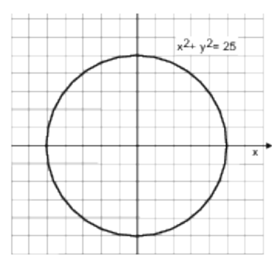
Define, evaluate, and compare functions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **functions, *y*-value, *x*-value, vertical line test, input, output, rate of change, linear function, non-linear function**

Common Core Standard	Unpacking What does this standard mean that a student will know and be able to do?
8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1] [1]Function notation is not required in Grade 8.	**8.F.1** Students understand rules that take x as input and gives y as output is a function. Functions occur when there is exactly one y-value is associated with any x-value. Using y to represent the output we can represent this function with the equations $y = x^2 + 5x + 4$. Students are **not** expected to use the function notation *f(x)* at this level. Students identify functions from equations, graphs, and tables/ordered pairs. **Graphs** Students recognize graphs such as the one below is a function using the vertical line test, showing that each x-value has only one y-value;



whereas, graphs such as the following are not functions since there are 2 y-values for multiple x-value.



	Tables or Ordered Pairs Students read tables or look at a set of ordered pairs to determine functions and identify equations where there is only one output (y-value) for each input (x-value). Functions **x** / **y**: 0, 3; 1, 9; 2, 27 Not A Function **x** / **y**: 16, 4; 16, -4; 25, 5; 25, -5 {(0, 2), (1, 3), (2, 5), (3, 6)} **Equations** Students recognize equations such as $y = x$ or $y = x^2 + 3x + 4$ as functions; whereas, equations such as $x^2 + y^2 = 25$ are not functions.
8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*	**8.F.2** Students compare two functions from different representations. Example 1: Compare the following functions to determine which has the greater rate of change. Function 1: y = 2x + 4 Function 2: **x** / **y**: -1, -6; 0, -3; 2, 3 *Solution*: The rate of change for function 1 is 2; the rate of change for function 2 is 3. Function 2 has the greater rate of change. Example 2: Compare the two linear functions listed below and determine which has a negative slope. Function 1: Gift Card Samantha starts with $20 on a gift card for the bookstore. She spends $3.50 per week to buy a magazine. Let y be the amount remaining as a function of the number of weeks, *x*. x / y: 0, 20; 1, 16.50; 2, 13.00; 3, 9.50

Functions:

x	**y**
0	3
1	9
2	27

Not A Function:

x	**y**
16	4
16	-4
25	5
25	-5

Example 1, Function 2:

x	**y**
-1	-6
0	-3
2	3

Example 2, Function 1:

x	y
0	20
1	16.50
2	13.00
3	9.50

Function 2: Calculator rental
The school bookstore rents graphing calculators for \$5 per month. It also collects a non-refundable fee of \$10.00 for the school year. Write the rule for the total cost (c) of renting a calculator as a function of the number of months (*m*).
$c = 10 + 5m$

Solution: Function 1 is an example of a function whose graph has a negative slope. Both functions have a positive starting amount; however, in function 1, the amount decreases 3.50 each week, while in function 2, the amount increases 5.00 each month.

NOTE: Functions could be expressed in standard form. However, the intent is not to change from standard form to slope-intercept form but to use the standard form to generate ordered pairs. Substituting a zero (0) for *x* and *y* will generate two ordered pairs. From these ordered pairs, the slope could be determined.

<u>Example 3:</u>
$2x + 3y = 6$

Let $x = 0$: $2(0) + 3y = 6$
$$3y = 6$$
$$\frac{3y}{3} = \frac{6}{3}$$
$$y = 2$$
Ordered pair: (0, 2)

Let $y = 0$: $2x + 3(0) = 6$
$$2x = 6$$
$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$
Ordered pair: (3, 0)

Using (0, 2) and (3, 0) students could find the slope and make comparisons with another function.

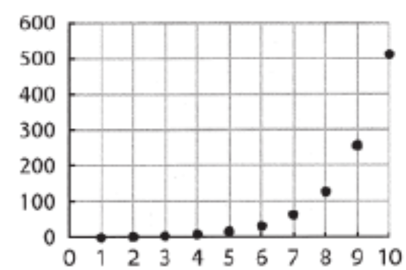
8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

8.F.3 Students understand that linear functions have a constant rate of change between any two points. Students use equations, graphs and tables to categorize functions as linear or non-linear.

<u>Example 1:</u>
Determine if the functions listed below are linear or non-linear. Explain your reasoning.

1. $y = -2x^2 + 3$
2. $y = 0.25 + 0.5(x - 2)$
3. $A = \pi r^2$
4.

X	Y
1	12
2	7
3	4
4	3
5	4
6	7

5. 

	Solution: 1. Non-linear 2. Linear 3. Non-linear 4. Non-linear; there is not a constant rate of change 5. Non-linear; the graph curves indicating the rate of change is not constant.

Functions 8.F

Common Core Cluster

Use functions to model relationships between quantities.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **linear relationship, rate of change, slope, initial value, y-intercept**

Common Core Standard	Unpacking What does this standard mean that a student will know and be able to do?
8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	**8.F.4** Students identify the rate of change (slope) and initial value (y-intercept) from tables, graphs, equations or verbal descriptions to write a function (linear equation). Students understand that the equation represents the relationship between the x-value and the y-value; what math operations are performed with the x-value to give the y-value. Slopes could be undefined slopes or zero slopes.

Tables:

Students recognize that in a table the y-intercept is the y-value when x is equal to 0. The slope can be determined by finding the ratio $\frac{y}{x}$ between the change in two y-values and the change between the two corresponding x-values.

<u>Example 1:</u>

Write an equation that models the linear relationship in the table below.

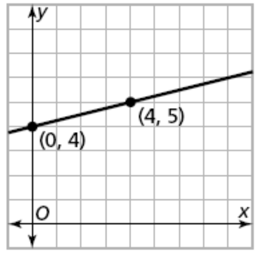
x	y
-2	8
0	2
1	-1

Solution: The y-intercept in the table below would be (0, 2). The distance between 8 and -1 is 9 in a negative direction → -9; the distance between -2 and 1 is 3 in a positive direction. The slope is the ratio of rise to run or $\frac{y}{x}$ or $\frac{-9}{3}$ = -3. The equation would be $y = -3x + 2$

Graphs:

Using graphs, students identify the y-intercept as the point where the line crosses the y-axis and the slope as the <u>rise.</u>
run

Example 2:
Write an equation that models the linear relationship in the graph below.



Solution: The y-intercept is 4. The slope is ¼ , found by moving up 1 and right 4 going from (0, 4) to (4, 5). The linear equation would be $y = \frac{1}{4}x + 4$.

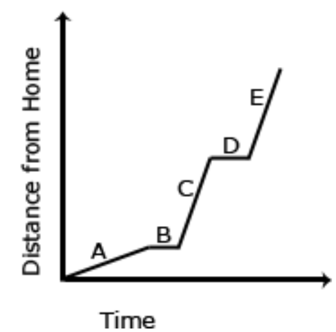
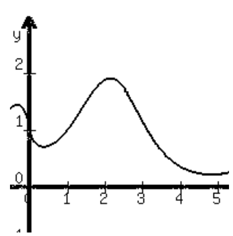
Equations:
In a linear equation the coefficient of x is the slope and the constant is the y-intercept. Students need to be given the equations in formats other than $y = mx + b$, such as $y = ax + b$ (format from graphing calculator), $y = b + mx$ (often the format from contextual situations), etc.

Point and Slope:
Students write equations to model lines that pass through a given point with the given slope.
Example 2:
A line has a zero slope and passes through the point (-5, 4). What is the equation of the line?
Solution: $y = 4$

Example 3:
Write an equation for the line that has a slope of ½ and passes though the point (-2, 5)
Solution: $y = \frac{1}{2}x + 6$
Students could multiply the slope ½ by the x-coordinate -2 to get -1. Six (6) would need to be added to get to 5, which gives the linear equation.

Students also write equations given two ordered pairs. **Note that point-slope form is not an expectation at this level.** Students use the slope and y-intercepts to write a linear function in the form $y = mx + b$.

Contextual Situations:
In contextual situations, the y-intercept is generally the starting value or the value in the situation when the independent variable is 0. The slope is the rate of change that occurs in the problem. Rates of change can often occur over years. In these situations it is helpful for the years to be "converted" to 0, 1, 2, etc. For example, the years of 1960, 1970, and 1980 could be represented as 0 (for 1960), 10 (for 1970) and 20 (for 1980).

	Example 4: The company charges \$45 a day for the car as well as charging a one-time \$25 fee for the car's navigation system (GPS). Write an expression for the cost in dollars, c, as a function of the number of days, d, the car was rented. *Solution:* $C = 45d + 25$ Students interpret the rate of change and the y-intercept in the context of the problem. In Example 3, the rate of change is 45 (the cost of renting the car) and that initial cost (the first day charge) also includes paying for the navigation system. Classroom discussion about one-time fees vs. recurrent fees will help students model contextual situations.
8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**8.F.5** Given a verbal description of a situation, students sketch a graph to model that situation. Given a graph of a situation, students provide a verbal description of the situation. Example 1: The graph below shows a John's trip to school. He walks to his Sam's house and, together, they ride a bus to school. The bus stops once before arriving at school. Describe how each part A – E of the graph relates to the story. *Solution:* A John is walking to Sam's house at a constant rate. B John gets to Sam's house and is waiting for the bus. C John and Sam are riding the bus to school. The bus is moving at a constant rate, faster than John's walking rate. D The bus stops. E The bus resumes at the same rate as in part C.  Example 2: Describe the graph of the function between $x = 2$ and $x = 5$?  *Solution:* The graph is non-linear and decreasing.

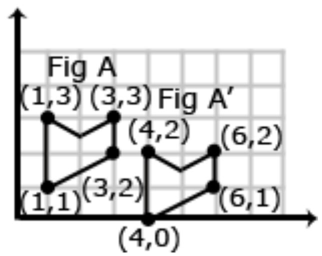
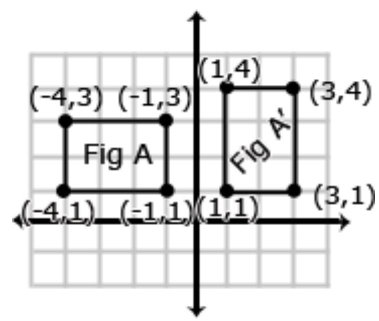
Geometry **8.G**

Common Core Cluster

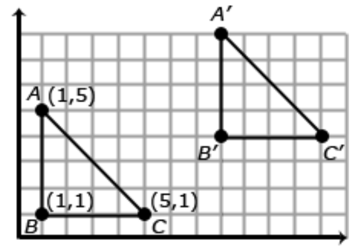
Understand congruence and similarity using physical models, transparencies, or geometry software.

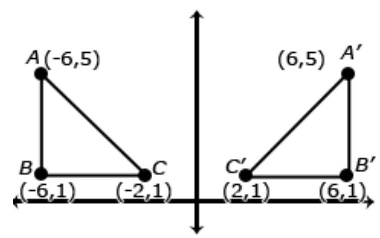
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **translations, rotations, reflections, line of reflection, center of rotation, clockwise, counterclockwise, parallel lines, betweenness, congruence, $\cong$, reading A' as "A prime", similarity, dilations, pre-image, image, rigid transformations, exterior angles, interior angles, alternate interior angles, angle-angle criterion, deductive reasoning, vertical angles, adjacent, supplementary, complementary, corresponding, scale factor, transversal, parallel**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
8.G.1 Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	**8.G.1** Students use compasses, protractors and rulers or technology to explore figures created from translations, reflections and rotations. Characteristics of figures, such as lengths of line segments, angle measures and parallel lines, are explored before the transformation (pre-image) and after the transformation (image). Students understand that these transformations produce images of exactly the same size and shape as the pre-image and are known as rigid transformations.

8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	**8.G.2** This standard is the students' introduction to congruency. Congruent figures have the same shape and size. Translations, reflections and rotations are examples of rigid transformations. A rigid transformation is one in which the pre-image and the image both have exactly the same size and shape since the measures of the corresponding angles and corresponding line segments remain equal (are congruent). Students examine two figures to determine congruency by identifying the rigid transformation(s) that produced the figures. Students recognize the symbol for congruency ($\cong$) and write statements of congruency. <u>Example 1:</u> Is Figure A congruent to Figure A'? Explain how you know.  *Solution:* These figures are congruent since A' was produced by translating each vertex of Figure A 3 to the right and 1 down <u>Example 2:</u> Describe the sequence of transformations that results in the transformation of Figure A to Figure A'.  *Solution:* Figure A' was produced by a 90º clockwise rotation around the origin.

<table>
<tr>
<td>8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</td>
<td>8.G.3 Students identify resulting coordinates from translations, reflections, and rotations (90º, 180º and 270º both clockwise and counterclockwise), recognizing the relationship between the coordinates and the transformation.

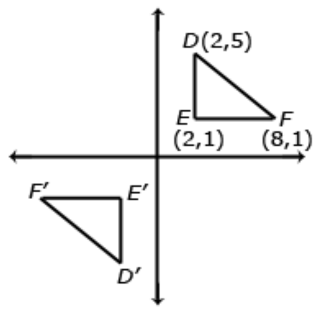
Translations
Translations move the object so that every point of the object moves in the same direction as well as the same distance. In a translation, the translated object is congruent to its pre-image. Triangle ABC has been translated 7 units to the right and 3 units up. To get from A (1,5) to A’ (8,8), move A 7 units to the right (from $x = 1$ to $x = 8$) and 3 units up (from $y = 5$ to $y = 8$). Points B and C also move in the same direction (7 units to the right and 3 units up), resulting in the same changes to each coordinate.


Reflections
A reflection is the “flipping” of an object over a line, known as the “line of reflection”. In the 8th grade, the line of reflection will be the x-axis and the y-axis. Students recognize that when an object is reflected across the y-axis, the reflected x-coordinate is the opposite of the pre-image x-coordinate (see figure below).


Likewise, a reflection across the x-axis would change a pre-image coordinate (3, -8) to the image coordinate of (3, 8) -- note that the reflected y-coordinate is opposite of the pre-image y-coordinate.

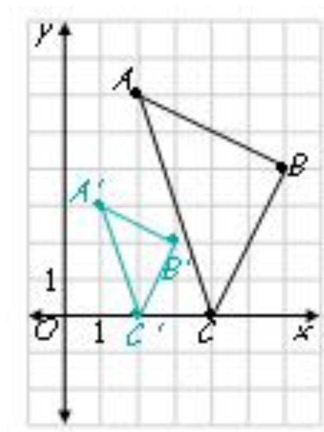
Rotations
A rotation is a transformation performed by “spinning” the figure around a fixed point known as the center of rotation. The figure may be rotated clockwise or counterclockwise up to 360º (at 8th grade, rotations will be around the origin and a multiple of 90º). In a rotation, the rotated object is congruent to its pre-image</td>
</tr>
</table>

Consider when triangle DEF is 180˚ clockwise about the origin. The coordinate of triangle DEF are D(2,5), E(2,1), and F(8,1). When rotated 180˚ about the origin, the new coordinates are D'(-2,-5), E'(-2,-1) and F'(-8,-1). In this case, each coordinate is the opposite of its pre-image (see figure below).



Dilations

A dilation is a non-rigid transformation that moves each point along a ray which starts from a fixed center, and multiplies distances from this center by a common scale factor. Dilations enlarge (scale factors greater than one) or reduce (scale factors less than one) the size of a figure by the scale factor. In 8th grade, dilations will be from the origin. The dilated figure is *similar* to its pre-image.

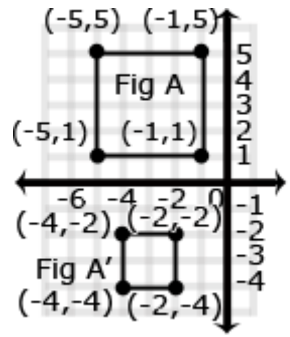
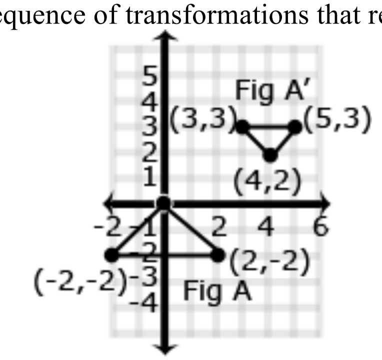


The coordinates of A are (2, 6); A' (1, 3). The coordinates of B are (6, 4) and B' are (3, 2). The coordinates of C are (4, 0) and C' are (2, 0). Each of the image coordinates is ½ the value of the pre-image coordinates indicating a scale factor of ½.

The scale factor would also be evident in the length of the line segments using the ratio: $\frac{\text{image length}}{\text{pre-image length}}$

Students recognize the relationship between the coordinates of the pre-image, the image and the scale factor for a dilation from the origin. Using the coordinates, students are able to identify the scale factor (image/pre-image).

Students identify the transformation based on given coordinates. For example, the pre-image coordinates of a triangle are A(4, 5), B(3, 7), and C(5, 7). The image coordinates are A(-4, 5), B(-3, 7), and C(-5, 7). What transformation occurred?

8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	**8.G.4** Similar figures and similarity are first introduced in the 8th grade. Students understand similar figures have congruent angles and sides that are proportional. Similar figures are produced from dilations. Students describe the sequence that would produce similar figures, including the scale factors. Students understand that a scale factor greater than one will produce an enlargement in the figure, while a scale factor less than one will produce a reduction in size. <u>Example1:</u> Is Figure A similar to Figure A'? Explain how you know.  *Solution*: Dilated with a scale factor of ½ then reflected across the *x*-axis, making Figures A and A' similar. Students need to be able to identify that triangles are similar or congruent based on given information. <u>Example 2:</u> Describe the sequence of transformations that results in the transformation of Figure A to Figure A'.  *Solution*: 90° clockwise rotation, translate 4 right and 2 up, dilation of ½. In this case, the scale factor of the dilation can be found by using the horizontal distances on the triangle (image = 2 units; pre-image = 4 units)

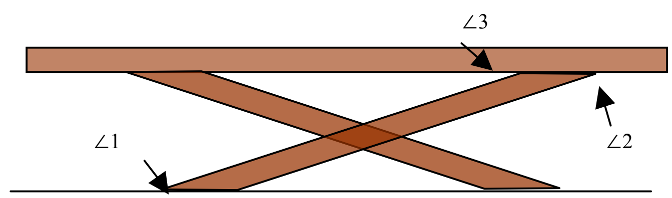
8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

8.G.5 Students use exploration and deductive reasoning to determine relationships that exist between the following: a) angle sums and exterior angle sums of triangles, b) angles created when parallel lines are cut by a transversal, and c) the angle-angle criterion for similarity of triangle.

Students construct various triangles and find the measures of the interior and exterior angles. Students make conjectures about the relationship between the measure of an exterior angle and the other two angles of a triangle. (the measure of an exterior angle of a triangle is equal to the sum of the measures of the other two interior angles) and the sum of the exterior angles (360º). Using these relationships, students use deductive reasoning to find the measure of missing angles.

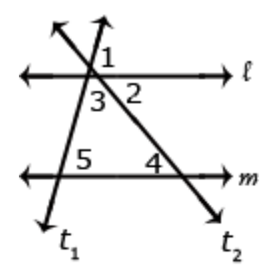
Students construct parallel lines and a transversal to examine the relationships between the created angles. Students recognize vertical angles, adjacent angles and supplementary angles from 7th grade and build on these relationships to identify other pairs of congruent angles. Using these relationships, students use deductive reasoning to find the measure of missing angles.

Example 1:
You are building a bench for a picnic table. The top of the bench will be parallel to the ground. If $m\angle 1 = 148^\circ$, find $m\angle 2$ and $m\angle 3$. Explain your answer.



Solution:
Angle 1 and angle 2 are alternate interior angles, giving angle 2 a measure of 148º. Angle 2 and angle 3 are supplementary. Angle 3 will have a measure of 32º so the $m\angle 2 + m\angle 3 = 180^\circ$

Example 2:
Show that $m\angle 3 + m\angle 4 + m\angle 5 = 180^\circ$ if line l and m are parallel lines and t_1 and t_2 are transversals.



Solution: $\angle 1 + \angle 2 + \angle 3 = 180^\circ$

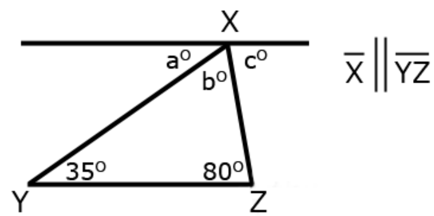
$\angle 5 \cong \angle 1$ corresponding angles are congruent therefore $\angle 1$ can be substituted for $\angle 5$

$\angle 4 \cong \angle 2$ alternate interior angles are congruent therefore $\angle 4$ can be substituted for $\angle 2$

Therefore $\angle 3 + \angle 4 + \angle 5 = 180^\circ$

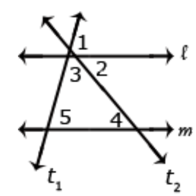
Students can informally conclude that the sum of the angles in a triangle is 180º (the angle-sum theorem) by applying their understanding of lines and alternate interior angles.

<u>Example 3:</u>
In the figure below Line *X* is parallel to Line $\overline{YZ}$. Prove that the sum of the angles of a triangle is 180º.



Solution: Angle *a* is 35º because it alternates with the angle inside the triangle that measures 35º. Angle *c* is 80º because it alternates with the angle inside the triangle that measures 80º. Because lines have a measure of 180º, and angles *a* + *b* + *c* form a straight line, then angle *b* must be 65 º → 180 – (35 + 80) = 65. Therefore, the sum of the angles of the triangle is 35º + 65 º + 80 º.

<u>Example 4:</u>
What is the measure of angle 5 if the measure of angle 2 is 45º and the measure of angle 3 is 60º?

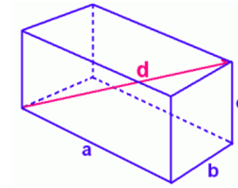
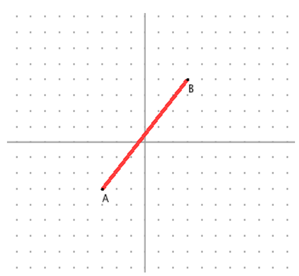
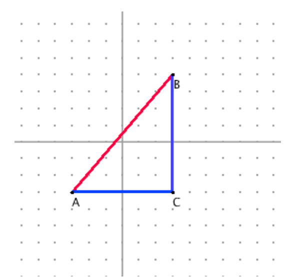


Solution: Angles 2 and 4 are alternate interior angles, therefore the measure of angle 4 is also 45º. The measure of angles 3, 4 and 5 must add to 180º. If angles 3 and 4 add to 105º the angle 5 must be equal to 75º.

Students construct various triangles having line segments of different lengths but with two corresponding congruent angles. Comparing ratios of sides will produce a constant scale factor, meaning the triangles are similar. Students solve problems with similar triangles.

Geometry	**8.G**
Common Core Cluster	
Understand and apply the Pythagorean Theorem.	
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **right triangle, hypotenuse, legs, Pythagorean Theorem, Pythagorean triple**	

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
8.G.6 Explain a proof of the Pythagorean Theorem and its converse.	**8.G.6** Using models, students explain the Pythagorean Theorem, understanding that the sum of the squares of the legs is equal to the square of the hypotenuse in a right triangle. Students also understand that given three side lengths with this relationship forms a right triangle. <u>Example 1:</u> The distance from Jonestown to Maryville is 180 miles, the distance from Maryville to Elm City is 300 miles, and the distance from Elm City to Jonestown is 240 miles. Do the three towns form a right triangle? Why or why not? *Solution*: If these three towns form a right triangle, then 300 would be the hypotenuse since it is the greatest distance. $180^2 + 240^2 = 300^2$ $32400 + 57600 = 90000$ $90000 = 90000$ ✓ These three towns form a right triangle.
8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	**8.G.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. <u>Example 1:</u> The Irrational Club wants to build a tree house. They have a 9-foot ladder that must be propped diagonally against the tree. If the base of the ladder is 5 feet from the bottom of the tree, how high will the tree house be off the ground? *Solution*: $a^2 + 5^2 = 9^2$ $a^2 + 25 = 81$ $a^2 = 56$ $\sqrt{a^2} = \sqrt{56}$ $a = \sqrt{56}$ or ~ 7.5 <u>Example 2:</u> Find the length of d in the figure to the right if $a = 8$ in., $b = 3$ in. and $c = 4$in.

	Solution: First find the distance of the hypotenuse of the triangle formed with legs *a* and *b*.  $8^2 + 3^2 = c^2$ $64^2 + 9^2 = c^2$ $73 = c^2$ $\sqrt{73} = \sqrt{c^2}$ $\sqrt{73}$ in. $= c$ The $\sqrt{73}$ is the length of the base of a triangle with *c* as the other leg and *d* is the hypotenuse. To find the length of *d*: $\sqrt{73}^2 + 4^2 = d^2$ $73 + 16 = d^2$ $89 = d^2$ $\sqrt{89} = \sqrt{d^2}$ $\sqrt{89}$ in. $= d$ Based on this work, students could then find the volume or surface area.
8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	**8.G.8** One application of the Pythagorean Theorem is finding the distance between two points on the coordinate plane. Students build on work from 6th grade (finding vertical and horizontal distances on the coordinate plane) to determine the lengths of the legs of the right triangle drawn connecting the points. Students understand that the line segment between the two points is the length of the hypotenuse. NOTE: The use of the distance formula is not an expectation. Example 1: Find the length of $\overline{AB}$.  *Solution:* 1. Form a right triangle so that the given line segment is the hypotenuse. 2. Use Pythagorean Theorem to find the distance (length) between the two points. $6^2 + 7^2 = c^2$ $36 + 49 = c^2$ $85 = c^2$ 

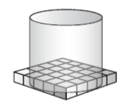
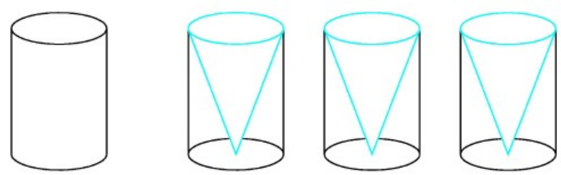
	Example 2: Find the distance between (-2, 4) and (-5, -6). *Solution*: The distance between -2 and -5 is the horizontal length; the distance between 4 and -6 is the vertical distance. Horizontal length: 3 Vertical length: 10 $10^2 + 3^2 = c^2$ $100 + 9 = c^2$ $109 = c^2$ $\sqrt{109} = \sqrt{c^2}$ $\sqrt{109} = c$ Students find area and perimeter of two-dimensional figures on the coordinate plane, finding the distance between each segment of the figure. (Limit one diagonal line, such as a right trapezoid or parallelogram)

Geometry 8.G

Common Core Cluster

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **cones, cylinders, spheres, radius, volume, height, Pi**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	**8.G.9** Students build on understandings of circles and volume from 7th grade to find the volume of cylinders, finding the area of the base πr^2 and multiplying by the number of layers (the height).   $V = \pi r^2 h$ find the area of the base and multiply by the number of layers Students understand that the volume of a cylinder is 3 times the volume of a cone having the same base area and height or that the volume of a cone is $\frac{1}{3}$ the volume of a cylinder having the same base area and height.  $V = \frac{1}{3}\pi r^2 h$ or $V = \frac{\pi r^2 h}{3}$ A sphere can be enclosed with a cylinder, which has the same radius and height of the sphere (Note: the height of the cylinder is twice the radius of the sphere). If the sphere is flattened, it will fill $\frac{2}{3}$ of the cylinder. Based on this model, students understand that the volume of a sphere is $\frac{2}{3}$ the volume of a cylinder with the same radius and height. The height of the cylinder is the same as the diameter of the sphere or $2r$. Using this information, the formula for the volume of the sphere can be derived in the following way:

$V = \pi r^2 h$ *cylinder volume formula*

$V = \frac{2}{3}\pi r^2 h$ *multiply by* $\frac{2}{3}$ *since the volume of a sphere is* $\frac{2}{3}$ *the cylinder's volume*

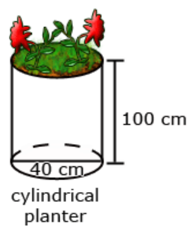
$V = \frac{2}{3}\pi r^2 2r$ *substitute 2r for height since 2r is the height of the sphere*

$V = \frac{4}{3}\pi r^3$ *simplify*

Students find the volume of cylinders, cones and spheres to solve real world and mathematical problems. Answers could also be given in terms of Pi.

<u>Example 1:</u>

James wanted to plant pansies in his new planter. He wondered how much potting soil he should buy to fill it. Use the measurements in the diagram below to determine the planter's volume.



Solution:

$V = \pi r^2 h$

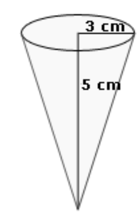
$V = 3.14\ (40)^2(100)$

$V = 502{,}400\ \text{cm}^3$

The answer could also be given in terms of π: $V = 160{,}000\ \pi$

<u>Example 2:</u>

How much yogurt is needed to fill the cone to the right? Express your answers in terms of Pi.



Solution:

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(3^2)(5)$

$V = \frac{1}{3}\pi\ (45)$

$V = 15\ \pi\ \text{cm}^3$

	<u>Example 3:</u> Approximately, how much air would be needed to fill a soccer ball with a radius of 14 cm? *Solution:* $V = \frac{4}{3}\pi r^3$ $V = \frac{4}{3}(3.14)(14^3)$ $V = 11.5 \text{ cm}^3$ "Know the formula" does not mean memorization of the formula. To "know" means to have an understanding of ***why*** the formula works and how the formula relates to the measure (volume) and the figure. This understanding should be for *all* students. **Note:** At this level composite shapes will not be used and only volume will be calculated.

Statistics and Probability — 8.SP

Common Core Cluster

Investigate patterns of association in bivariate data.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **bivariate data, scatter plot, linear model, clustering, linear association, non-linear association, outliers, positive association, negative association, categorical data, two-way table, relative frequency**

Common Core Standard	Unpacking What does this standard mean that a student will know and be able to do?
8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	**8.SP.1** Bivariate data refers to two-variable data, one to be graphed on the *x*-axis and the other on the *y*-axis. Students represent numerical data on a scatter plot, to examine relationships between variables. They analyze scatter plots to determine if the relationship is linear (positive, negative association or no association) or non-linear. Students can use tools such as those at the National Center for Educational Statistics to create a graph or generate data sets. (http://nces.ed.gov/nceskids/createagraph/default.aspx) Data can be expressed in years. In these situations it is helpful for the years to be "converted" to 0, 1, 2, etc. For example, the years of 1960, 1970, and 1980 could be represented as 0 (for 1960), 10 (for 1970) and 20 (for 1980).

Example 1:

Data for 10 students' Math and Science scores are provided in the chart. Describe the association between the Math and Science scores.

Student	1	2	3	4	5	6	7	8	9	10
Math	64	50	85	34	56	24	72	63	42	93
Science	68	70	83	33	60	27	74	63	40	96

Solution: This data has a positive association.

Example 2:

Data for 10 students' Math scores and the distance they live from school are provided in the table below. Describe the association between the Math scores and the distance they live from school.

Student	1	2	3	4	5	6	7	8	9	10
Math	64	50	85	34	56	24	72	63	42	93
Distance from School (miles)	0.5	1.8	1	2.3	3.4	0.2	2.5	1.6	0.8	2.5

Solution: There is no association between the math score and the distance a student lives from school.

Example 3:
Data from a local fast food restaurant is provided showing the number of staff members and the average time for filling an order are provided in the table below. Describe the association between the number of staff and the average time for filling an order.

Number of Staff	3	4	5	6	7	8
Average time to fill order (seconds)	56	24	72	63	42	93

Solution: There is a positive association.

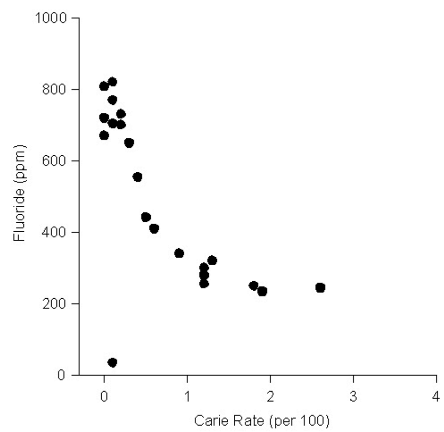
Example 4:
The chart below lists the life expectancy in years for people in the United States every five years from 1970 to 2005. What would you expect the life expectancy of a person in the United States to be in 2010, 2015, and 2020 based upon this data? Explain how you determined your values.

Date	1970	1975	1980	1985	1990	1995	2000	2005
Life Expectancy (in years)	70.8	72.6	73.7	74.7	75.4	75.8	76.8	77.4

Solution: There is a positive association.

Students recognize that points may be away from the other points (outliers) and have an effect on the linear model.
NOTE: Use of the formula to identify outliers is **not** expected at this level.

Students recognize that not all data will have a linear association. Some associations will be non-linear as in the example below:



8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line

8.SP.2 Students understand that a straight line can represent a scatter plot with linear association. The most appropriate linear model is the line that comes closest to most data points. The use of linear regression is not expected. If there is a linear relationship, students draw a linear model. Given a linear model, students write an equation.

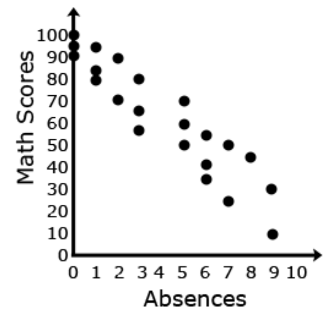
8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

8.SP.3 Linear models can be represented with a linear equation. Students interpret the slope and y-intercept of the line in the context of the problem.

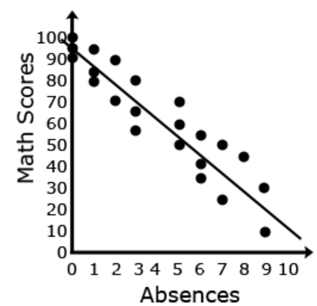
<u>Example 1:</u>

1. Given data from students' math scores and absences, make a scatterplot.

Absences	Math Scores
3	65
5	50
1	95
1	85
3	80
6	34
5	70
3	56
0	100
7	24
8	45
2	71
9	30
0	95
6	55
6	42
2	90
0	92
5	60
7	50
9	10
1	80



2. Draw a linear model paying attention to the closeness of the data points on either side of the line.



3. From the linear model, determine an approximate linear equation that models the given data (about $y = -\frac{25}{3}x + 95$)

4. Students should recognize that 95 represents the y-intercept and $-\frac{25}{3}$ represents the slope of the line. In the context of the problem, the y-intercept represents the math score a student with 0 absences could expect. The slope indicates that the math scores decreased 25 points for every 3 absences.

<table>
<tr>
<td></td>
<td>5. Students can use this linear model to solve problems. For example, through substitution, they can use the equation to determine that a student with 4 absences should expect to receive a math score of about 62. They can then compare this value to their line.</td>
</tr>
<tr>
<td>8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?</td>
<td>8.SP.4 Students understand that a two-way table provides a way to organize data between two categorical variables. Data for both categories needs to be collected from each subject. Students calculate the relative frequencies to describe associations.

<u>Example 1:</u>
Twenty-five students were surveyed and asked if they received an allowance and if they did chores. The table below summarizes their responses.

<table>
<tr><th></th><th>Receive Allowance</th><th>No Allowance</th></tr>
<tr><td>Do Chores</td><td>15</td><td>5</td></tr>
<tr><td>Do Not Do Chores</td><td>3</td><td>2</td></tr>
</table>

Of the students who do chores, what percent do not receive an allowance?
Solution: 5 of the 20 students who do chores do not receive an allowance, which is 25%</td>
</tr>
</table>

We would like to acknowledge the Arizona Department of Education for allowing us to use some of their examples and graphics.